

The asymptotic safety scenario in quantum gravity hypothesizes that the quantum field theory of the spacetime metric possesses a nontrivial ultraviolet fixed point for the renormalization group flow. In other words, the quantum laws of spacetime geometry approach a scale-invariant regime when examined at microscopic length scales. I will discuss the role of random geometry and critical phenomena in the search for concrete models of scale-invariant quantum geometry, highlighting recent mathematical advances in lower-dimensional toy models as well as the mathematical challenges on the route to three- and four-dimensional models.